Zero-Coupon Yield Curve Construction Methodology

Bond Pricing & Product Development Department

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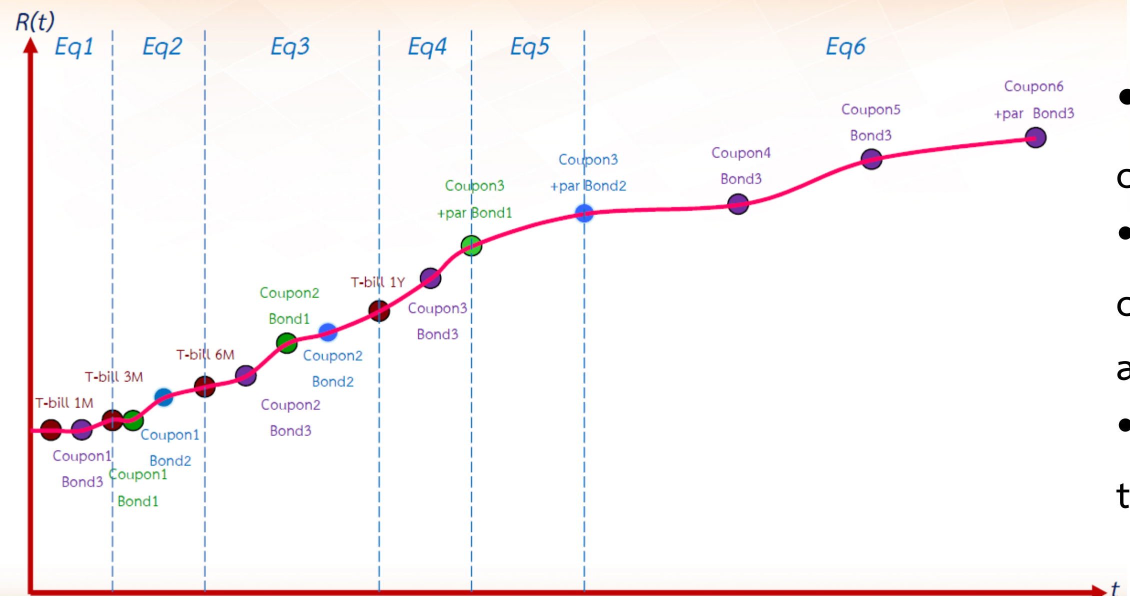
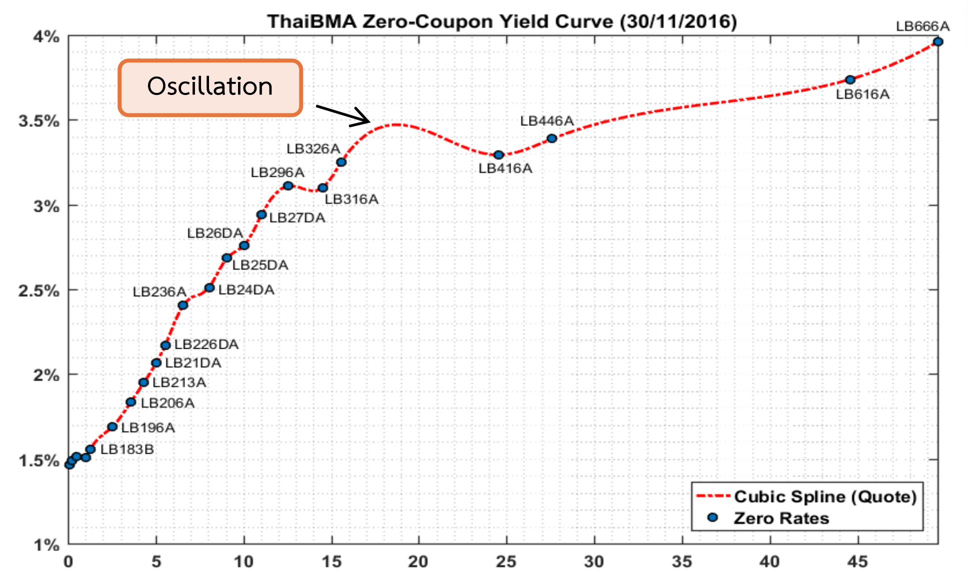
How the curve is created



- Decompose all coupon payments into zero coupon bonds.
- Generate equations for all the yield points corresponding to the coupon dates using of an interpolation approach.
- We use Cubic Hermite Spline Interpolation to connect all points together.

Background of Rational Criteria



To construct zero-coupon yield curve which has <u>less oscillation when the data is not smooth.</u>

Cubic Spline Interpolation → Cubic Hermite Spline Interpolation

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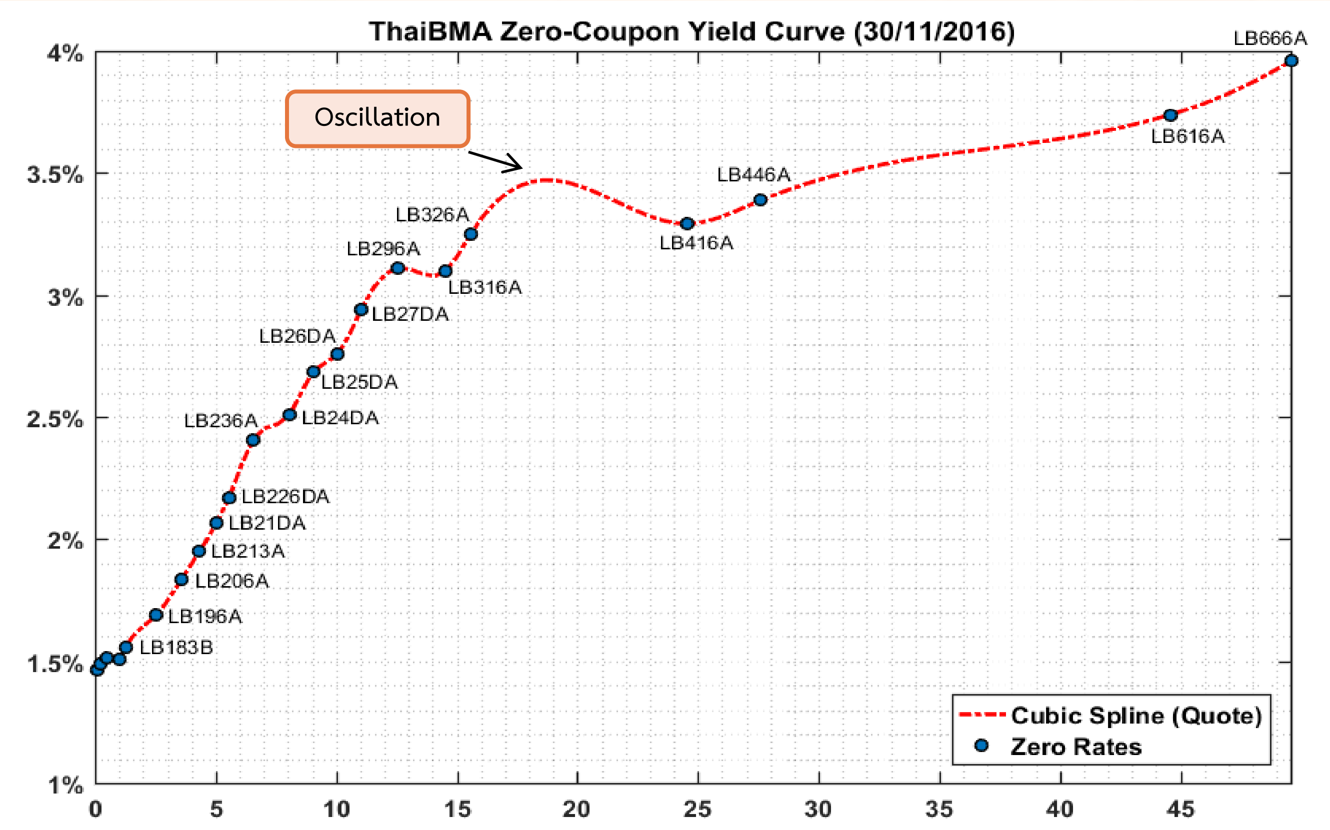
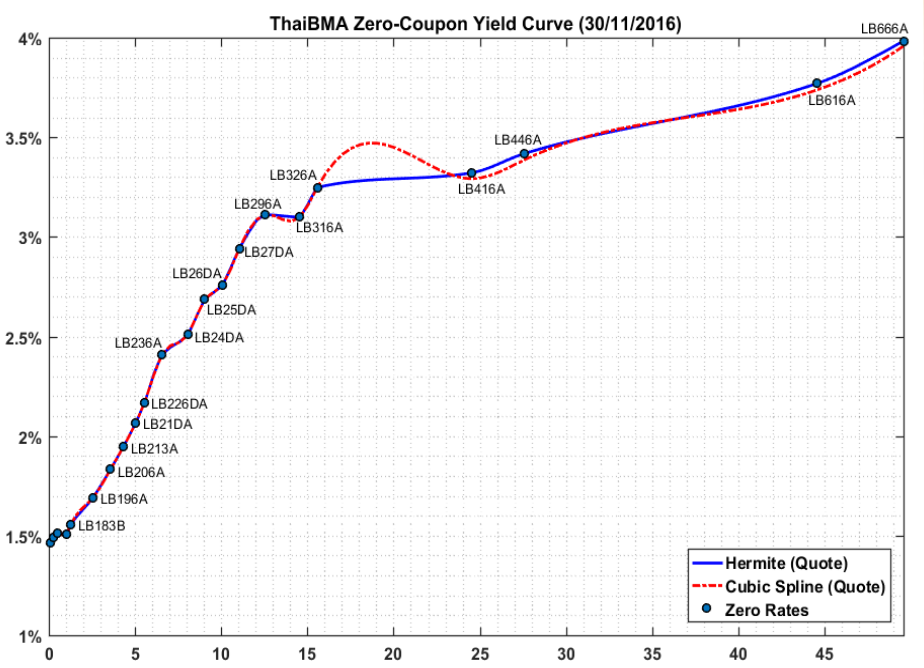
Problem of Cubic Spline Interpolation



Introduce Hermite Method



When the data is not smooth, Cubic Hermite Spline interpolation has no overshoots and less oscillation. Since, second order derivative is not required continuous.

Cubic Spline Interpolation

Suppose $t_1, t_2, \ldots, t_n$ *(TTM)* and $r_1, r_2, \ldots, r_n$ *(spot rate)* are known.

Each *i*, we have coefficients (a_i, b_i, c_i, d_i) for $1 \le i \le n-1$

$$r_i(t) = a_i(t - t_i)^3 + b_i(t - t_i)^2 + c_i(t - t_i) + d_i \qquad t_i \le t \le t_{i+1}$$

Note that

$$r_i'(t) = 3a_i(t - t_i)^2 + 2b_i(t - t_i) + c_i \qquad t_i \le t \le t_{i+1}$$

$$r_i''(t) = 6a_i(t - t_i) + 2b_i \qquad t_i \le t \le t_{i+1}$$

Requirement:

- $r_i(t)$ is continuous, so $r_{i-1}(t_i) = r_i(t_i)$
- $r'_i(t)$ is continuous, so $r'_{i-1}(t_i) = r'_i(t_i)$
- $r''_i(t)$ is continuous, so $r''_{i-1}(t_i) = r''_i(t_i)$

Cubic Hermite Spline Interpolation

Suppose $t_1, t_2, \ldots, t_n$ *(TTM)* and $r_1, r_2, \ldots, r_n$ *(spot rate)* are known.

Each i, we have coefficients (a_i, b_i, c_i, d_i) for $1 \le i \le n-1$

$$r_i(t) = a_i(t-t_i)^3 + b_i(t-t_i)^2 + c_i(t-t_i) + d_i \qquad t_i \le t \le t_{i+1}$$

Note that

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$$r_i''(t) = 6a_i(t-t_i) + 2b_i \qquad t_i \le t \le t_{i+1}$$

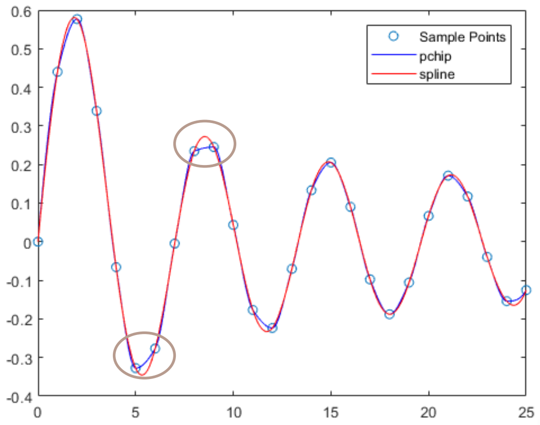
Condition:

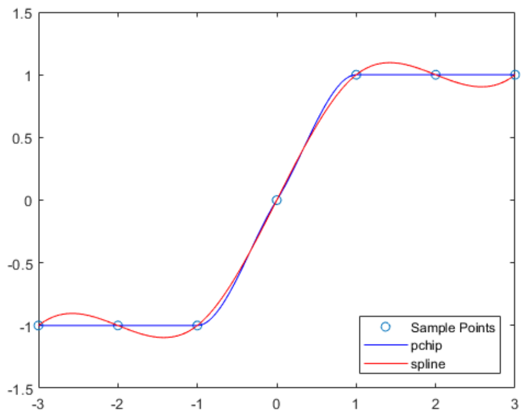
- $r_i(t)$ and $r'_i(t)$ is continuous, but $r''_i(t)$ is not needed to be continuous.
- The value of c_i for $1 < i < n$ are chosen to be the slope at t_i that pass through (t_j, r_j) for $j = i-1, i, i+1$
- For instance, c_1 is chosen to be the slope at t_1 that passes through (t_j, r_j) for $j = 1, 2, 3$ and c_n is chosen likewise.

Cubic Spline vs Hermite Interpolation

Cubic Spline (Old)

- Cubic Spline produces a smoother result, such that S"(x) is continuous.
- Cubic Spline produces a more accurate result if the data consists of values of a smooth function.





Hermite Spline (New)

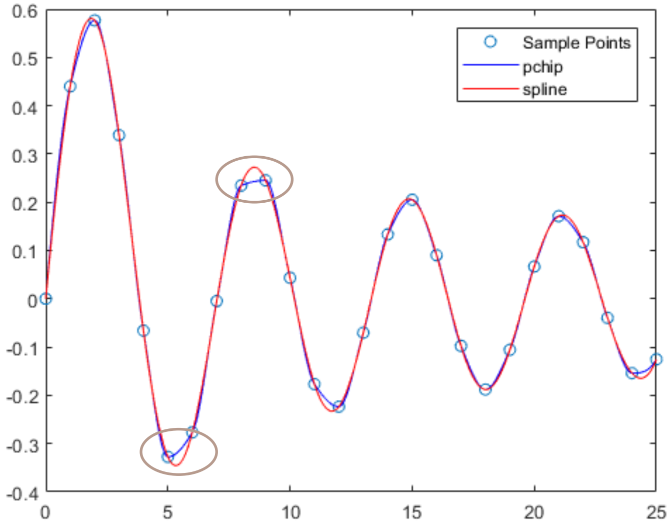
- Cubic Hermite Spline has <u>no overshoots and less oscillation</u> when the data is not smooth.
- Cubic Hermite Spline is less expensive to set up.

Cubic Spline vs Hermite Interpolation (1)

Cubic spline constructs S(x) in almost the same way cubic Hermite constructs P(x). However, cubic spline chooses the slopes at the t_j differently, namely to make S"(x) continuous.

This differences has several effects.

- Cubic spline produces a smoother result, such that S"(x) is continuous.
- Cubic spline produces a more accurate result if the data consists of values of a smooth function.



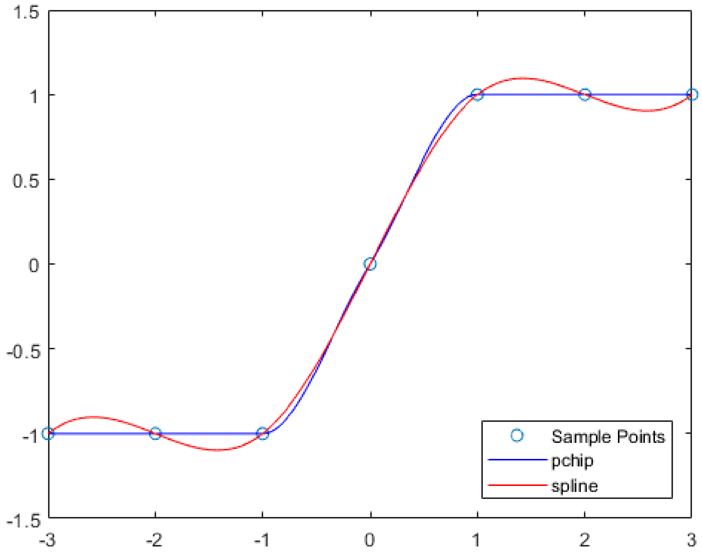
Source: https://www.mathworks.com/help/matlab/ref/pchip.html

Cubic Spline vs Hermite Interpolation (2)

Cubic spline constructs S(x) in almost the same way cubic Hermite constructs P(x). However, cubic spline chooses the slopes at the t_j differently, namely to make S"(x) continuous.

This differences has several effects.

- Cubic Hermite has <u>no overshoots and less oscillation</u> when the data is not smooth.
- Cubic Hermite is less expensive to set up.



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